AFRL-RB-WP-TR-2008-3164

PROBABILITY-OF-OPPORTUNITY FOR AIRBORNE SEARCH PROBLEMS

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Advanced Structural Concepts Branch
Structures Division

OCTOBER 2008
Final Report

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AIR FORCE RESEARCH LABORATORY
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UNITED STATES AIR FORCE

NOTICE AND SIGNATURE PAGE

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REPORT DOCUMENTATION PAGE

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
October 2008	Final	01 January 2006 – 30 September 2008

4. TITLE AND SUBTITLE

PROBABILITY-OF-OPPORTUNITY FOR AIRBORNE SEARCH PROBLEMS

5a. CONTRACT NUMBER
In-house

5b. GRANT NUMBER

5c. PROGRAM ELEMENT NUMBER
0602201

6. AUTHOR(S)

Jason Bowman and Jeff Dubois

5d. PROJECT NUMBER
A0AF

5e. TASK NUMBER

5f. WORK UNIT NUMBER
0A

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Advanced Structural Concepts Branch (AFRL/RBSA)
Structures Division, Air Force Research Laboratory
Air Vehicles Directorate
Wright-Patterson Air Force Base, OH 45433-7542
Air Force Materiel Command, United States Air Force

8. PERFORMING ORGANIZATION REPORT NUMBER
AFRL-RB-WP-TR-2008-3164

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Air Force Research Laboratory
Air Vehicles Directorate
Wright-Patterson Air Force Base, OH 45433-7542
Air Force Materiel Command
United States Air Force

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL/RBSA

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-RB-WP-TR-2008-3164

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

13. SUPPLEMENTARY NOTES
PAO Case Number: 88ABW-2008-0673, 23 Oct 2008. Report contains color.

14. ABSTRACT

The concept of probability-of-opportunity grew out of work being done under the Air Force Research Laboratory's Next Generation Unmanned Aircraft System (AFRL/NGUAS) efforts. Specifically, AFRL was looking to better understand the relationships between sensor, sensor platform, and target characteristics in order to effectively search an area or route. Time-based simulation is quite costly and does not give immediate insight into what drives the search problem due to the lack of analytic relationships. The key to finding an efficient simulation technique was to break the search or find function of the system into two parts: 1) the probability that the sensor is looking at the right place at the right time (opportunity) and 2) given the former, the probability of the sensor in detecting the target. It was possible to develop an analytic approach to the opportunity problem using basic statistics.

15. SUBJECT TERMS

UAS, medium altitude, multimission, ISR, force application, systems engineering

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
a. REPORT Unclassified	**b. ABSTRACT** Unclassified	**c. THIS PAGE** Unclassified	SAR	50	Jason Bowman **19b. TELEPHONE NUMBER** *(Include Area Code)* N/A

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GLOSSARY

$A_{footprint}$	Area of the sensor footprint, estimated by $l_{eff} \times w_{eff}$
A_{search}	Area of the search area given by $L \times W$
A_i	A subdivided portion of the search area
a, b, c, d	Points along the P_{opp} probability density function curve
CDF	Cumulative density function
E	Expected value
F	Component function of the P_{opp} CDF
g	Acceleration due to gravity
L	Dimension of the search area in the primary direction of the search. See also W.
l_{eff}	Effective depth or length of a sensor footprint. See also w_{eff}.
N_{find}	Number of targets found in a given search area
$N_{targets}$	Number of targets in a given search area P Probability
n	Sustainable load factor in a turn
n_{swath}	Number of passes to completely search an area
P_d	Sensor Probability-of-Detect given that a target is in the field-of-view
P_{find}	System Probability-of-Find or -Detect
P_{opp}	Probability-of-Opportunity to detect
PDF	Probability density function
R	Turn radius
T	Period of time
T_{dwell}	Dwell time of a sensor field-of-view on a given point in space
T_{leg}	Total time spent in straight-and-level flight for a unit search
$T_{revisit}$	Time to complete one search cycle
T_{target}	Target availability, i.e. period of time a target is available to be detected
T_{turn}	Total time spent in turning flight for a unit search
t	Starting point in time
t_{dwell}	Time at the leading edge of the sensor footprint
t_{target}	Time when a given target becomes available for detection
V_{patrol}	Patrol speed of a sensor platform
W	Dimension of the search area perpendicular to the primary direction of the search. See also L
w_{eff}	Effective width of a sensor footprint. See also l_{eff}

Z Variable substitution for $t_{dwell} - t_{target}$

Δ Variable substituation for $T_{target} - T_{dwell}$

$\Delta\psi$ Initial change of heading required for a teardrop turn

1. INTRODUCTION

The concept of Probability-of-Opportunity grew out of work being done under the Air Force Research Laboratory's (AFRL) Next Generation UAS (NGUAS) efforts, an MQ-9 follow-on. Specifically, AFRL was looking to better understand the relationships between sensor, sensor platform, and target characteristics in order to effectively search an area or route. Actual time-based simulation is quite costly (for this level of effort) and does not give immediate insight into what drives the search problem due to the lack of analytic relationships.

The key to finding an efficient simulation technique was to break the search or find function of the system into two parts. The ability to find something depends on 1) the probability that the sensor is looking at the right place at the right time (opportunity) and 2) given the former, the probability of the sensor in detecting the target. It was possible to develop an analytic approach to the opportunity problem using basic statistics. When coupled with analytic approaches to the detect problem, a rapid method of accomplishing first order system (vehicles + sensors + targets) search trades was possible.

The opportunity method presented is a very general approach to the problem and is applicable across a broad class of sensors and targets. However, because the approach is so general, the problem space needs to be reduced to a few simple parameters, and, in this reduction, some basic assumptions of the method may be violated. The details will be discussed later, but it is believed these are second order effects and easily correctible with adjustment factors.

2. THE AREA OR LINE SEARCH PROBLEM SPACE

The fundamental problem space is depicted in Figure 1. There is an area or route to be searched with length L and width W. Multiple sensor footprints (multiple search platforms or multiple sensors per platform) search the area according to some search pattern. Targets appear randomly for a period of time, called the Target Availability, and with a density dictated by the specific mission. Targets may also move, although for the analytic problem they are required to be stationary.
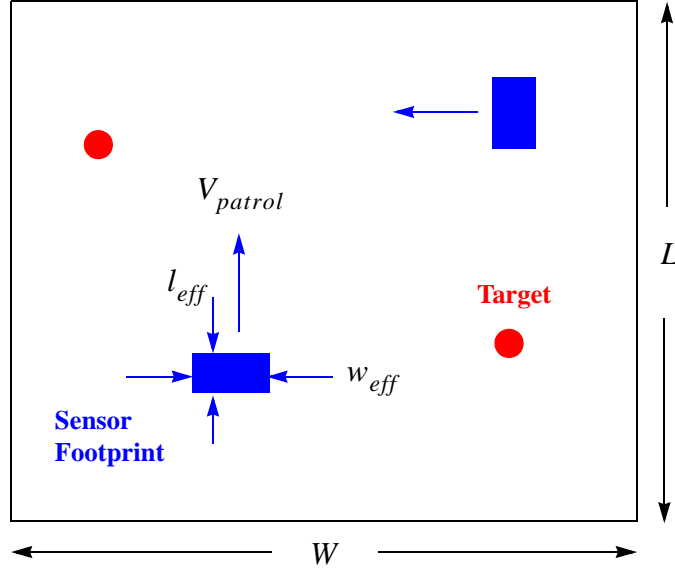
Figure 1: Basic Killbox Properties

The goal is to find a quick method for determining the System Probability-of-Detection or -Find. As stated in the Introduction, this can be decomposed into two parts that can be evaluated separately.

$$P_{find} = P_{opp}P_d \qquad (1)$$

where

P_{find}	System Probability-of-Find or -Detection
P_{opp}	Probability that the sensor is looking at the right place at the right time
P_d	Sensor Probability-of-Detection given that the sensor is looking at the right place at the right time

What is meant by the sensor looking at the right place at the right time? This is where the target characteristics come into play. The target is located at some point in space (and is perhaps

moving) for a finite period of time. The period of time is defined as the Target Availability (T_{target}). If the sensor footprint encompasses the target when it is available, there is an opportunity to detect.

Instead of thinking about the problem spatially, it was discovered that a temporal approach was the best way to generalize the opportunity problem. The size and shape of the search area does not matter explicitly using the temporal approach.

There are three periods of time required to accomplish the opportunity derivation.

$T_{revisit}$ The time it takes to complete one search cycle.

T_{dwell} Time that the sensor footprint dwells over any point on the ground. This is the effective length or depth l_{eff} of the footprint divided by the patrol speed V_{patrol}.

T_{target} Target Availability, i.e. the length of time the target is available to be detected.

Graphically, these periods of times are illustrated in Figure 2. Note that T refers to a period of time whereas t refers to a starting point in time. Both will be required in the opportunity derivation.
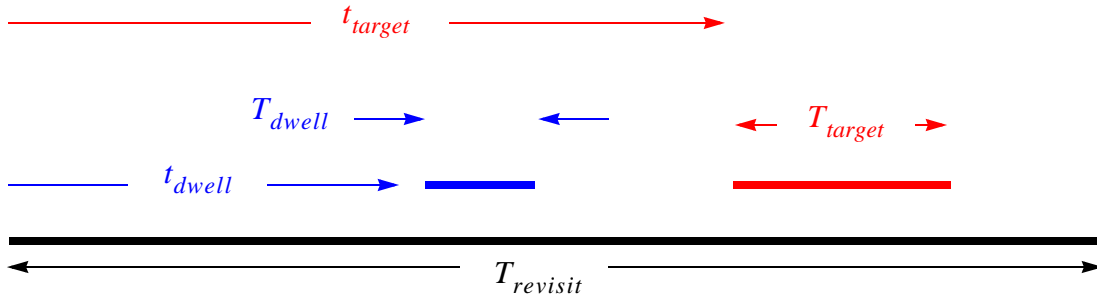
Figure 2: Timeline of Events

The time when the target first appears, t_{target}, and the position in time of the sensor footprint, t_{dwell}, are classified as uniformly random variables. The time that a target is potentially in the sensor footprint, T_{dwell}, and the target availability, T_{target}, are constants determined from the sensor (footprint and patrol speed) and target characteristics.

Note that the opportunity analysis developed here does not address multiple patrol cycles, so the aircraft is assumed to be transported back to the starting point. However, only one patrol cycle is

required to determine the best search parameters. Cycles where targets do not appear produce a null set $P_{opp} \equiv 0$, and when Target Availabilities exceed the time for one cycle, $P_{opp} \equiv 1$.

An opportuntiy is defined as any overlap of the sensor and the target time envelopes. Mathematically, this is the joint probability of the two conditions depicted in Figure 3. Essentially there is no opportunity when the target appears and disappears before the sensor can get to it, and there is no opporunity when the target appears after the sensor passes. The joint probability of these two conditions is the Probability-of-Opportunity.
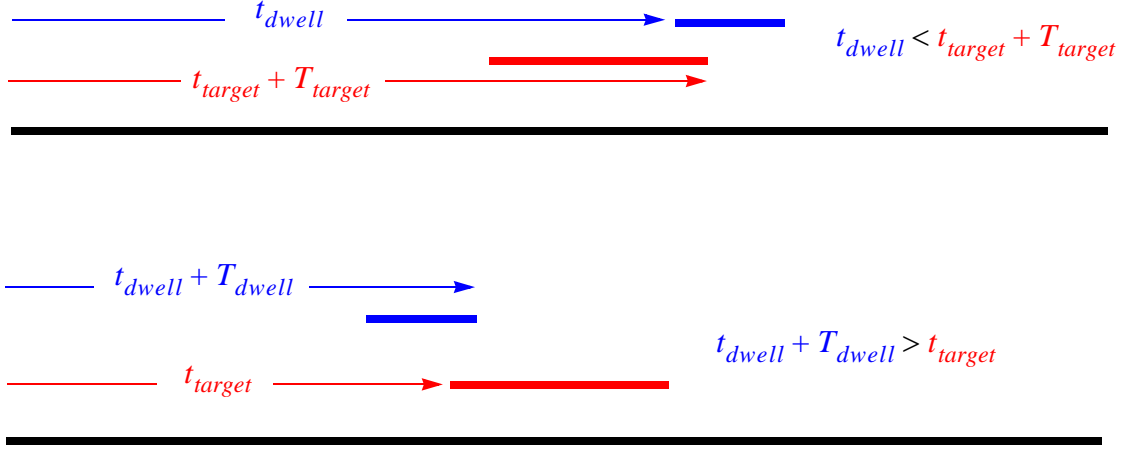
Figure 3: Conditions for an Opportunity

$$t_{dwell} < t_{target} + T_{target}$$
$$t_{dwell} + T_{dwell} > t_{target}$$

(2)

Note that

$$0 < t_{dwell} < T - T_{dwell}$$
$$0 < t_{target} < T - T_{target}$$

(3)

Equations 3 ensure that the left and right edges of the sensor and target start times do not go outside the cycle bounds.

From equations 2, collecting the random variables on the left side of the inequalities yields

$$t_{dwell} - t_{target} < T_{target}$$
$$t_{dwell} - t_{target} > -T_{dwell}$$

(4)

4

The negative of dwell time is always negative since dwell time is always positive, and since target availability can never be negative, equations 4 can be combined and arranged as follows

$$-T_{dwell} < t_{dwell} - t_{target} < T_{target} \tag{5}$$

For simplicity define

$$Z \equiv t_{dwell} - t_{target} \tag{6}$$

The probability of the condition in equation 5 occuring is then

$$P[-T_{dwell} < Z < T_{target}] = P[Z < T_{target}] - P[Z < -T_{dwell}] \tag{7}$$

The first probability is the probability that the sensor starts looking before the target becomes unavailable (goes away) for detection. The second is the probability that the sensor is just close enough to catch the target when it first appears. Although this sounds like the original statement, this is the rigorous mathematical result. The difference of these probabilities is the Probability-of-Opportunity.

To further the development, an understanding of the distrubtion of $Z \equiv t_{dwell} - t_{target}$ is required. First, the range of Z is

$$
\begin{aligned}
Z_{min} &= min(t_{dwell}) - max(t_{target}) \\
Z_{max} &= max(t_{dwell}) - min(t_{target})
\end{aligned}
\tag{8}
$$

Since

$$
\begin{aligned}
min(t_{dwell}) &= min(t_{target}) = 0 \\
max(t_{dwell}) &= T_{revisit} - T_{dwell} \\
max(t_{target}) &= T_{revisit} - T_{target}
\end{aligned}
\tag{9}
$$

the range of Z is more explicitly given as

$$
\begin{aligned}
Z_{min} &= -(T_{revisit} - T_{target}) \\
Z_{max} &= T_{revisit} - T_{dwell}
\end{aligned}
\tag{10}
$$

Simple simulation over 100,000 runs demonstrates that the PDF of Z is generally trapezoidal and degenerates to a triangle when $T_{dwell} = T_{target}$ as illustrated in Figure 4.

$$T_{target} \neq T_{dwell}$$
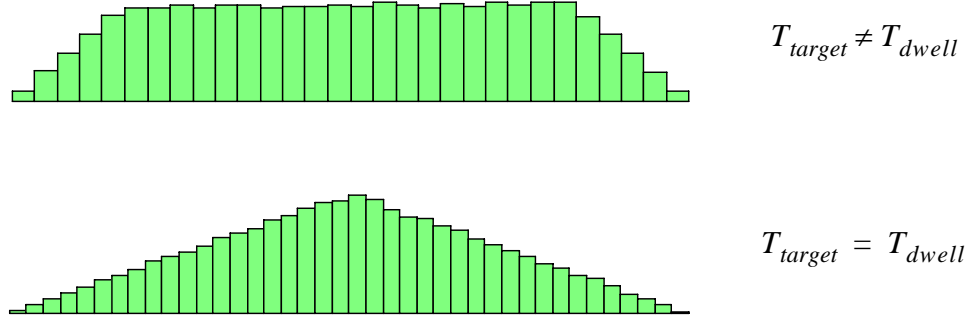
$$T_{target} = T_{dwell}$$

Figure 4: Histograms of Z

Figure 5 illustrates in more detail the geometry of the distribution. Equations 10 give the left and right limits of the distribution. The length of the top of the distribution is the range between $max(t_{dwell})$ and $max(t_{target})$, which is $|\Delta| = |T_{target} - T_{dwell}|$. Points b and c are then easily derived. The height is calculated by integrating the PDF since the CDF must equal one (1) at the extreme positive value of Z.

$$h = \cfrac{1}{T_{revisit} - \cfrac{T_{dwell} + T_{target}}{2} + \cfrac{|\Delta|}{2}}$$

$$|\Delta| = |T_{target} - T_{dwell}|$$

$$b = \frac{\Delta}{2} - \frac{|\Delta|}{2}$$

$$c = \frac{\Delta}{2} + \frac{|\Delta|}{2}$$

$$E[Z] = \frac{\Delta}{2}$$

$$a = -(T_{revisit} - T_{target})$$

$$d = T_{revisit} - T_{dwell}$$

Figure 5: PDF of Z

6
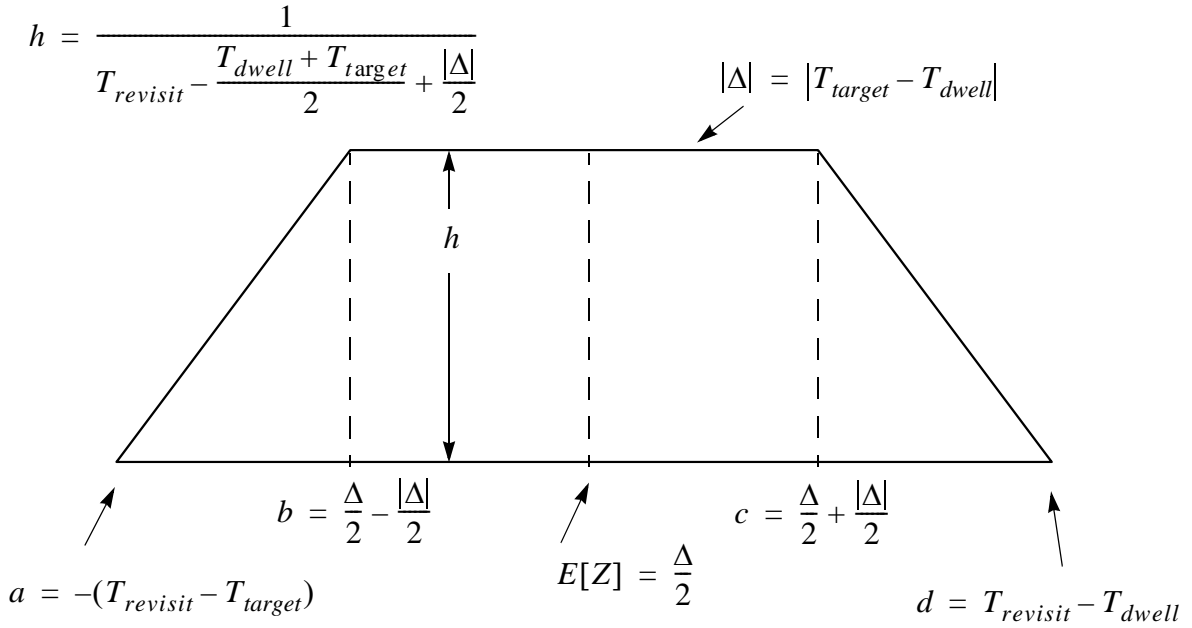
In summary, the PDF parameters are

$$\Delta \equiv T_{target} - T_{dwell}$$

$$a \equiv -(T_{revisit} - T_{target})$$

$$b \equiv \frac{\Delta}{2} - \frac{|\Delta|}{2}$$

$$c \equiv \frac{\Delta}{2} + \frac{|\Delta|}{2} \tag{11}$$

$$d \equiv T_{revisit} - T_{dwell}$$

$$h \equiv \frac{1}{T_{revisit} - \dfrac{T_{dwell} + T_{target}}{2} + \dfrac{|\Delta|}{2}}$$

The PDF can now be expressed as

$$f(Z) = \begin{cases} 0 & Z < a \\ \dfrac{h}{b-a}(Z-a) & a \le Z \le b \\ h & b < Z \le c \\ \dfrac{h}{d-c}(d-Z) & c < Z \le d \\ 0 & d < Z \end{cases} \tag{12}$$

The CDF is then

$$F(Z) = \begin{cases} 0 & Z < a \\ \dfrac{h}{b-a}\dfrac{(Z-a)^2}{2} & a \le Z \le b \\ F(b) + h(Z-b) & b < Z \le c \\ F(c) + \dfrac{1}{2}\dfrac{h}{d-c}[(d-c)^2 - (d-Z)^2] & c < Z \le d \\ 1 & d < Z \end{cases} \tag{13}$$

7

Recall that

$$P_{opp} = P[Z < T_{target}] - P[Z < -T_{dwell}] \tag{14}$$

Which yields

$$P_{opp} = F(T_{target}) - F(-T_{dwell}) \tag{15}$$

Equation 15 is plotted in Figure 6 using the two non-dimensional parameters $\dfrac{T_{target}}{T_{revisit}}$ and $\dfrac{T_{dwell}}{T_{revisit}}$.

Figure 6: Probability of Opportunity as a Function of Target and Sensor Availability

From Figure 6, it can be seen that equation 15 ultimately has a fairly simple approximate form
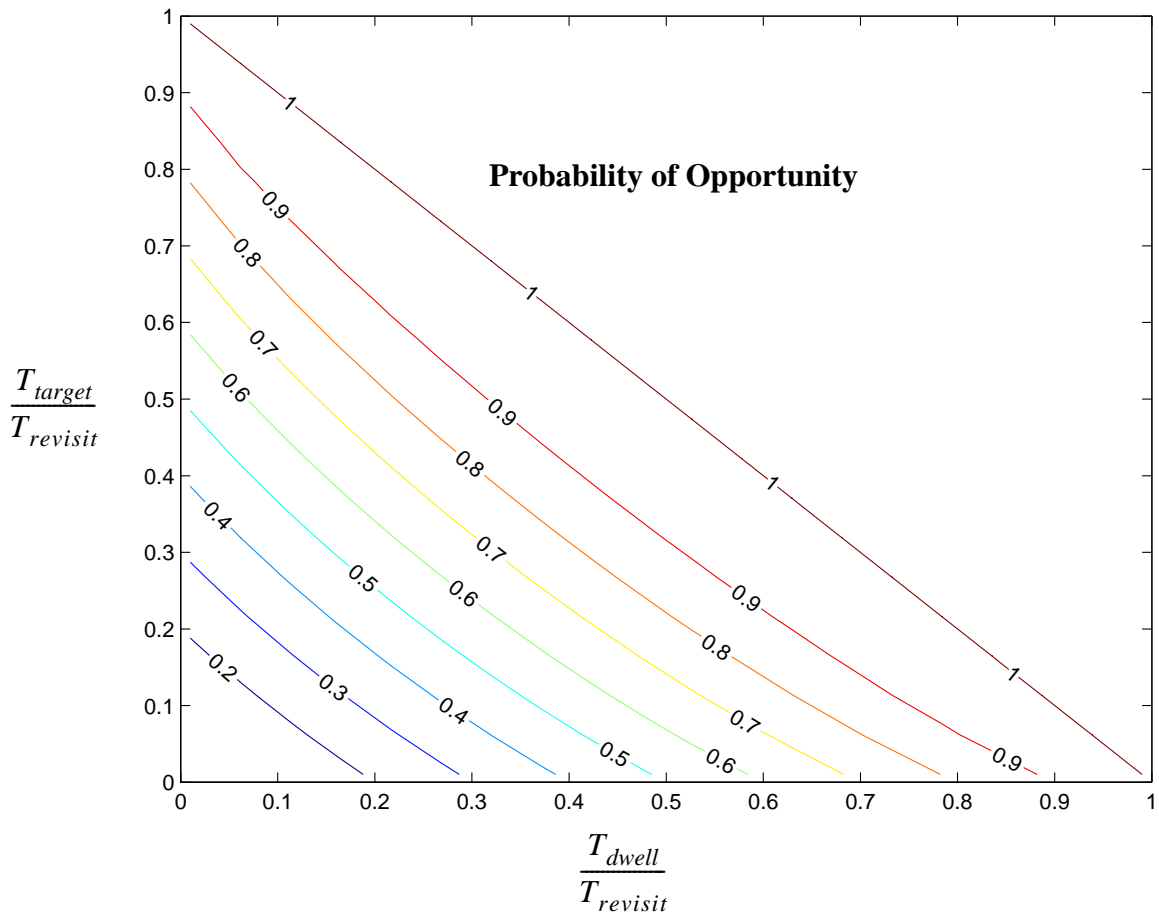
$$
P_{opp} = \begin{cases} \dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} & \dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} \leq 1 \\[4ex] 1 & \dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} > 1 \end{cases} \tag{16}
$$

although it is not immediately obvious by inspection of equation 13, from which this result was derived. The analytic solution is developed for the case when $T_{target} > T_{dwell}$ (due to the absolute value in equations 11), which is the typical case for air search where the sensor dwell time is relatively small compared to the target availability.

The first step is to determine if simplication of equation 15 yields a result close to equation 16 with suitable eliminations or approximations. The result is

$$
P_{opp} = \begin{cases} \dfrac{1}{1 - \dfrac{T_{dwell}}{T_{revisit}}} \left(\dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} - \dfrac{1}{1 - \dfrac{T_{target}}{T_{revisit}}} \left(\dfrac{T_{dwell}}{T_{revisit}} \right)^2 \right) & \dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} \leq 1 \\[5ex] 1 & \dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} > 1 \end{cases} \tag{17}
$$

Unfortunately, this is not close to the approximate function. However, if we add and subtract by $\dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}}$ in equation 17,

$$
P_{opp} = \dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} + e \tag{18}
$$

where

$$
e = -\dfrac{T_{target}}{T_{revisit}} - \dfrac{T_{dwell}}{T_{revisit}} + \dfrac{1}{1 - \dfrac{T_{dwell}}{T_{revisit}}} \left(\dfrac{T_{target}}{T_{revisit}} + \dfrac{T_{dwell}}{T_{revisit}} - \dfrac{1}{1 - \dfrac{T_{target}}{T_{revisit}}} \left(\dfrac{T_{dwell}}{T_{revisit}} \right)^2 \right) \tag{19}
$$

By inspection of Figure 6, the error is greatest along the diagonal where $\dfrac{T_{target}}{T_{revisit}} = \dfrac{T_{dwell}}{T_{revisit}}$. This error is plotted in Figure 7, and it can be seen that the error does not exceed 0.09 or 9% and in many cases is half that or less, especially off the diagonal where the error is usually 1-2%.

$$\frac{T_{target} = T_{dwell}}{T_{revisit}}$$
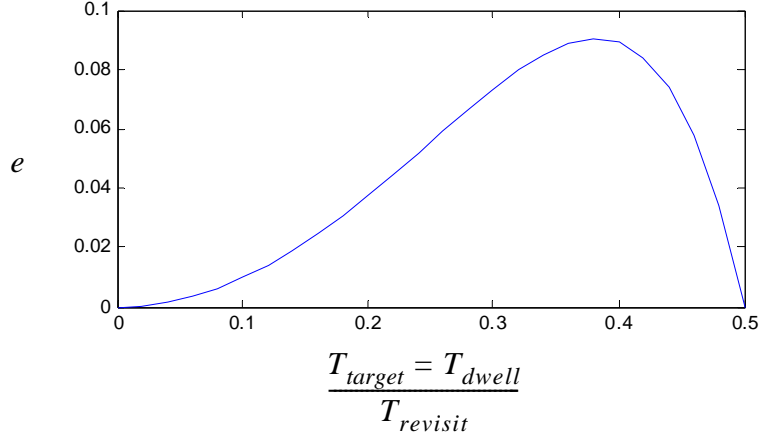
Figure 7: Maximum (Diagonal) Error in the Approximate Opportunity Function

From the approximate probability-of-opportunity and by inspection of Figure 6,

$$P_{opp} \approx \frac{T_{target}}{T_{revisit}} + \frac{T_{dwell}}{T_{revisit}} \tag{20}$$

it is easy to deduce how to improve detection opportunities. Dwell time can be increased, but this comes at the expense of reduced resolution, which then limits detections if an opportunity exists, and reduced patrol speed, which can reduce opportunities. The most effective way to improve opportunities is to reduce the revisit time. This is explored in the remainder of this document and can include different patrol patterns, flying faster, higher load factors in turns, and multiple vehicles and sensors.

3. TRANSLATING A SEARCH INTO OPPORTUNITY PARAMETERS

In order to conduct the opportunity analysis, the actual search problem needs to be translated into opportunity parameters. Recall that the opportunity analysis requires three fundamental time measurements — Revisit Period $T_{revisit}$, Sensor Dwell Time T_{dwell}, and Target Availability T_{target}. Target Availability is dictated by the mission and type of target. The Revisit Period and Sensor Dwell Time, on the other hand, are complicated functions of other more basic inputs. The purpose of this section is to develop expressions for these three quantities so that opportunities can be evaluated.

Sensor Dwell Time is the effective length or depth (in the direction of travel) of the footprint divided by the patrol speed.

$$T_{dwell} = \frac{l_{eff}}{V_{patrol}} \tag{21}$$

The effective length or depth is required since the sensor footprint usually does not have a rectangular shape. For an optical sensor, the footprint is usually a trapezoid, and for a synthetic aperture radar it is an area bounded by two circular arcs. This effective footprint length or depth is the first place where the opportunity assumptions begin to break down.

The Revisit Period is a function of patrol pattern, patrol speed, sensor footprint width, and the dimensions of the search area. The width of the footprint determines how many swaths are required to cover an area once, the patrol speed determines how fast that area is covered and the time spent in turns, and the patrol pattern can also determine the effective number of swaths and how the vehicle turns.

Rather than thinking in terms of swaths, the $T_{revisit}$ expression is easier to develop in terms of unit searches. Then the revisit period for any patrol pattern can be expressed as

$$T_{revisit} = n_{search} T_{leg} \left(1 + \frac{T_{turn}}{T_{leg}} \right) \tag{22}$$

where T_{leg} is the total time spent in straight and level flight for a unit search and T_{turn} is the total time spent in turning flight for a unit search.

Two basic search patterns, illustrated in Figure 8, are examined here and translated into opportunity inputs. Additional search patterns are left to the reader.
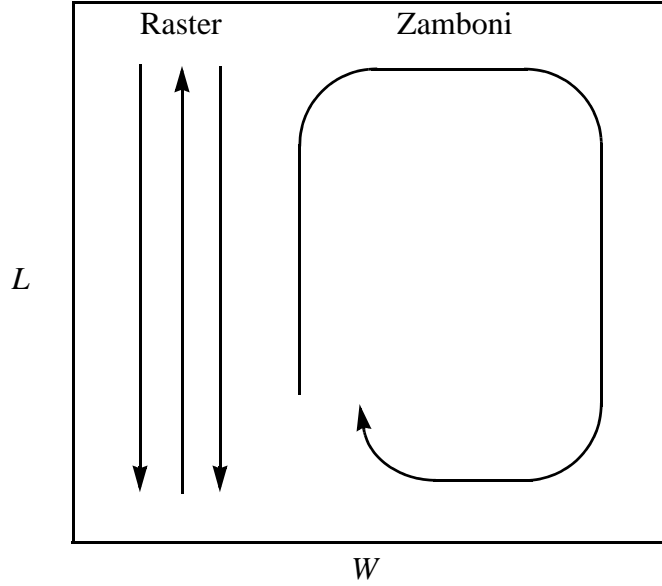
Figure 8: Basic Search Patterns

The most basic search is the raster search. In the raster search, the sensors flies up and down the search area moving left or right an amount equal to the effective footprint width, w_{eff}, when the boundaries of the search area are reached. In the raster search, aircraft turning physics are ignored as is the distance that would have to be flown between search columns. However, the patrol speed is considered so that the raster search takes a finite and realistic amount of time.

The natural unit of the raster search is one straight leg followed by a turn. The number of patrol legs or units, n_{search}, or columns in the raster search is the width of the patrol area divided by the sensor footprint width, rounded upwards.

$$n_{search} = ceil\left(\frac{W}{w_{eff}}\right) \tag{23}$$

The unit straight leg time for the raster search is simply

$$T_{straight} = \frac{L}{V_{patrol}} \tag{24}$$

$T_{turn} = 0$ for the basic raster search, which ignores the physical turns at the end of each leg.

The raster search can be modified to include physically realizable turns at the end of each search leg. However, the way the aircraft can turn is greatly influenced by the patrol speed, sustainable

load factor, and effective width of the footprint as shown in Figure 9. If the turn diameter exceeds the effective footprint width, the aircraft must make a teardrop turn initially away from the next patrol leg. If the turn diameter is less than the effective footprint width, the aircraft must execute a quarter turn, followed by a straight leg, then another quarter turn. Only when the turn diameter and footprint width are the same does the aircraft fly a simple circular arc. Although by no means the global optimum, the time spent in the turn is minimized when the vehicle parameters are selected to produce a circular turn. It will be seen later that there is a worst case patrol speed for a given target availability due to the effect of the time spent in turns.
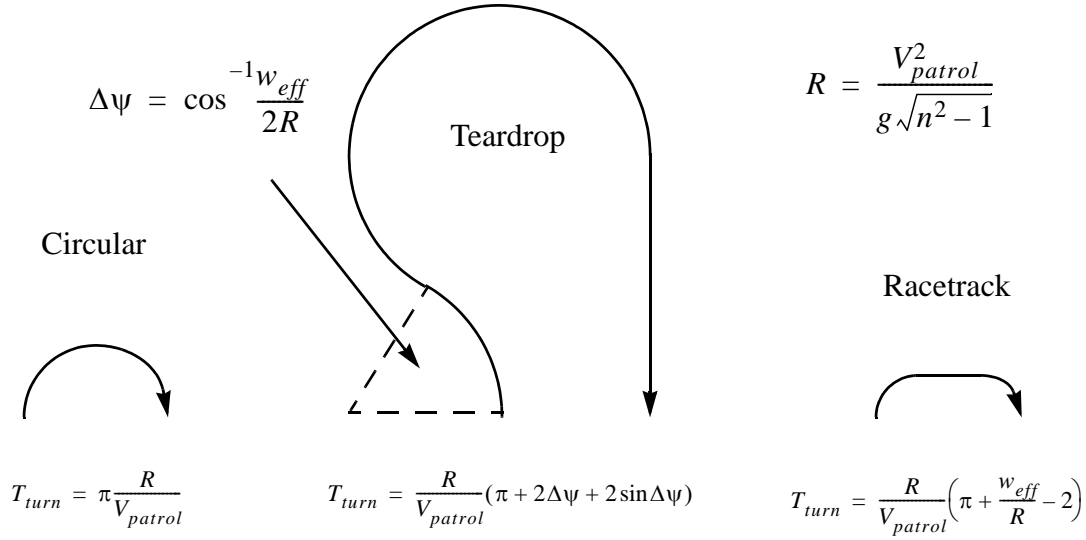
$$\Delta\psi \ = \ \cos^{-1}\frac{w_{eff}}{2R}$$

Teardrop

$$R \ = \ \frac{V_{patrol}^2}{g\sqrt{n^2-1}}$$

Circular

Racetrack

$$T_{turn} \ = \ \pi\frac{R}{V_{patrol}}$$

$$T_{turn} \ = \ \frac{R}{V_{patrol}}(\pi + 2\Delta\psi + 2\sin\Delta\psi)$$

$$T_{turn} \ = \ \frac{R}{V_{patrol}}\left(\pi + \frac{w_{eff}}{R} - 2\right)$$

Figure 9: Raster Search Turns and Time-to-Turn

Another basic search pattern developed is the Zamboni. This pattern is useful for synthetic aperture radars that typically look perpendicular to the flight path. This pattern is also useful when a raster turn would be so wide that opportunities are missed. The problem with the Zamboni is that transition paths from one patrol column to another are revisited at a much higher rate than interior points of the search. See Figure 10. This introduces inefficiencies in the search, which may have to be accepted for a variety of reasons, and it violates the basic assumption regarding Probability-of-Opportunity that revisit rate is uniform across the search area. It may be possible to decompose the opportunity problem so that different opportunity analyses are done in different parts of the search area such that the revisit rate is uniform for a given analysis. However, it is currently assumed that ignoring the non-uniform revist rate does not introduce gross errors into the opportunity analysis.
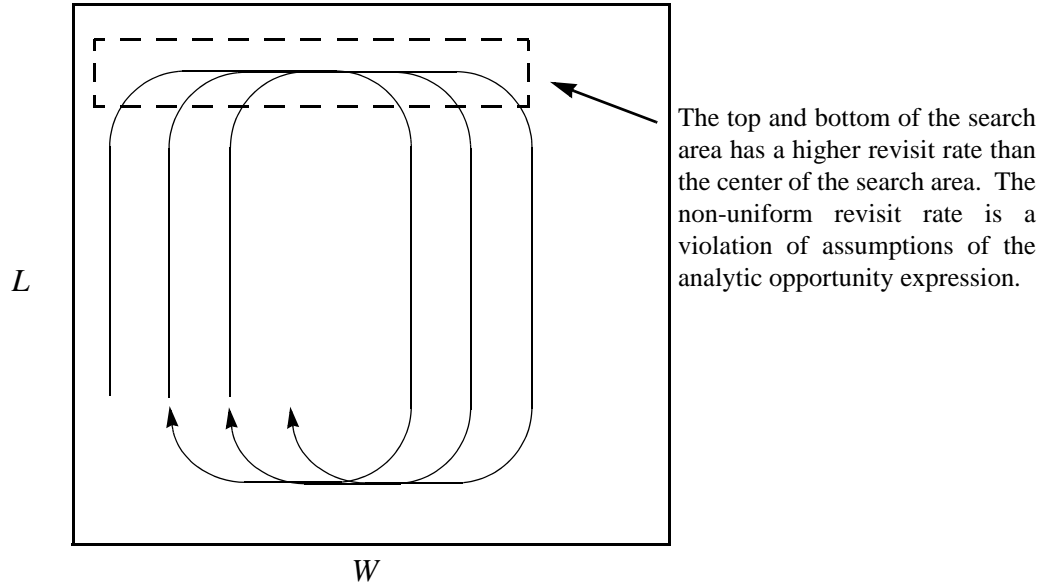
Figure 10: Non-Uniform Revisit Rate of the Zamboni Search Pattern

The top and bottom of the search area has a higher revisit rate than the center of the search area. The non-uniform revisit rate is a violation of assumptions of the analytic opportunity expression.

For the Zamboni, the natural unit search pattern is one circuit, so the number of unit searches is approximately half of that for the raster search. However, the search times for one unit are about twice as long. Table 1 summarizes the parameters for the Raster and Zamboni search patterns.

Table 1: Number of Passes, Leg and Turn Times for Raster and Zamboni Searches

	Raster	**Zamboni**
n_{search}	$ceil\left(\dfrac{W}{w_{eff}}\right)$	$ceil\left(\dfrac{W/2}{w_{eff}}\right)$
$T_{straight}$	$\dfrac{L}{V_{patrol}}$	$\dfrac{2L + 2(W/2 - 2R) - w_{eff}}{V_{patrol}}$
T_{turn}	$a\dfrac{R}{V_{patrol}}$ circle: $a = \pi$ teardrop: $a = \pi + 2\Delta\psi + 2\sin\Delta\psi$ racetrack: $a = \pi + \dfrac{w_{eff}}{R} - 2$	$2\pi\dfrac{R}{V_{patrol}}$

If the *ceiling* function is ignored, the revisit period for either the raster or Zamboni search can be written as

$$T_{revisit} = \frac{W}{w_{eff}} \frac{L}{V_{patrol}} \left(1 + a\frac{R}{L} + \frac{b}{L} \right) \tag{25}$$

where $b = 0$ for raster patterns. From Table 1,

$$\frac{b}{L} = \frac{1}{2} \frac{1}{L/W} - 2\frac{R}{L} - \frac{1}{2} \left(\frac{1}{W/w_{eff}} \right) \left(\frac{1}{L/W} \right) \tag{26}$$

The dwell-to-revisit period ratio, which is one component of the P_{opp} expression, can now be expressed as

$$\frac{T_{dwell}}{T_{revisit}} = \frac{l_{eff}/V_{patrol}}{\dfrac{W}{w_{eff}} \dfrac{L}{V_{patrol}} \left(1 + a\dfrac{R}{L} + \dfrac{b}{L} \right)} \tag{27}$$

$$\frac{T_{dwell}}{T_{revisit}} = \frac{l_{eff} w_{eff}}{LW} \frac{1}{1 + a\dfrac{R}{L} + \dfrac{b}{L}} \tag{28}$$

$$\frac{T_{dwell}}{T_{revisit}} = \frac{A_{footprint}}{A_{search}} \frac{1}{1 + a\dfrac{R}{L} + \dfrac{b}{L}} \tag{29}$$

Equation 29 makes it clear that to increase the Probability-of-Opportunity, the sensor footprint size should be increased relative to the size of the search area and search pattern overhead (time spent in turns for all patterns and repositioning legs for the Zamboni) need to be reduced. These results are obvious. However, there is now a mathematical relationship that can be explored.

Finally, there is the Target Availability, which is a given. Since an expression for the Revisit Period has already been developed, the Target Availability-to-Revisit Period ratio is directly computed.

4. PROBABILITY-OF-OPPORTUNITY ANALYSIS

In the previous section, the search problem was cast in terms of the two contributing parameters to the Probability-of-Opportunity, $\frac{T_{dwell}}{T_{revisit}}$ and $\frac{T_{target}}{T_{revisit}}$. The purpose of this section is to explore the behavior of the Probability-of-Opportunity relationship.

Note that equation 29 suggests that several non-dimensional parameters control the dwell time ratio including $\frac{A_{footprint}}{A_{search}}$, $\frac{R}{L}$, $\frac{w_{eff}}{2R}$ (from the a coefficient), and $\frac{W}{w_{eff}}$ and $\frac{L}{W}$ (both from the b coefficient). Although the meaning of $\frac{R}{L}$ and $\frac{w_{eff}}{2R}$ is clear, in an expanded form (eq. 30 below), it is unclear how to group the parameters that can be controlled such as patrol speed and sustainable load factor in any meaningful way. The only non-dimensional choice is $\frac{V_{patrol}^2}{gL}$ and n. However, what does $\frac{V_{patrol}^2}{gL}$ mean physically?

$$\frac{R}{L} = \frac{V_{patrol}^2}{gL\sqrt{n^2 - 1}} \qquad (30)$$

Because of the difficulty in choosing meaningful non-dimensional parameters, the dwell and availability time ratios will be computed from dimensional inputs, which are more meaningful.

The first and simplest analysis that can be done is to understand the relative magnitudes of $\frac{T_{dwell}}{T_{revisit}}$ and $\frac{T_{target}}{T_{revisit}}$. Assume a 1 km footprint depth and a Mach 0.1 patrol speed. Dwell Time is approximately thirty (30) seconds. At Mach 0.4, this is reduced to approximately eight (8) seconds. The lower bound for Target Availabilities is about five (5) minutes for current operations, and a lower bound of ten (10) minutes is more typical. Even for a very fleeting target (5 minutes) and MQ-1 patrol speeds, $\frac{T_{target}}{T_{revisit}} \gg \frac{T_{dwell}}{T_{revisit}}$ by at least an order of magnitude.

Therefore, Probability-of-Opportunity can be further approximated as

$$P_{opp} \approx \frac{T_{target}}{T_{revisit}} \qquad (31)$$

At this point, realistic values for the search parameters need to be chosen to understand the values and/or ranges that achieve a high number of detection opportunities. Table 2 lists the range of parameters that will be considered.

Table 2: Probability-of-Opportunity Study Parameters

Parameter	Range
Search Area	100, 400, 1600 km^2
Target Availability	10 minutes (P_{opp} for other availabilities will simply be a multiple of this value).
Mach	0.1 to 0.8
Sustained Load Factor	1.035 - 2.0 (15º - 60º bank)
Search Area Aspect Ratio	1,10
Search Pattern	Raster (No Turns), Raster, Zamboni
Footprint Area	1 km^2 (1 km by 1 km), 4 km^2 (2 km by 2 km)

In Figure 11, the Probability-of-Opportunity for three search patterns is compared for various area (square) search sizes. Although opportunities greater than unity are shown, opportunity can never exceed unity. The following conclusions can be made:

- The turns for a real search pattern can reduce the opportunities significantly compared to the raster (no search) ideal.
- Raster is better at lower speeds, but Zamboni is significantly better at higher speeds.
- Zamboni searches are relatively insensitive to sustained load factor. Raster is relatively insensitive to load factor at low speeds only.
- Raster searches require speed and load factor increases to improve opportunities. Zambonis just require speed to improve opportunities (assuming the Zamboni pattern is flyable at higher speeds).

In Figure 12, the three search patterns are compared for various route search sizes with the following conclusions:

- Zamboni is not good for route searches because the wide Zamboni turn can not be completed except at low speeds.
- Raster searches with greater speed are better for routes.

Figures 13 and 14 illustrate the differences between raster for area and route searches and Zamboni for area and route searches, respectively. These reinforce the point that Zambonis with speed are better for areas and rasters with speed are better for routes. However, this is intuitive. Figures 17-28 illustrate more of the raw data.

Recall that the Target Availability was set at a fairly agressive ten (10) minutes. If the Target Availability increases to twenty (20) minutes, the opportunities double. That may change the choice of search parameters.
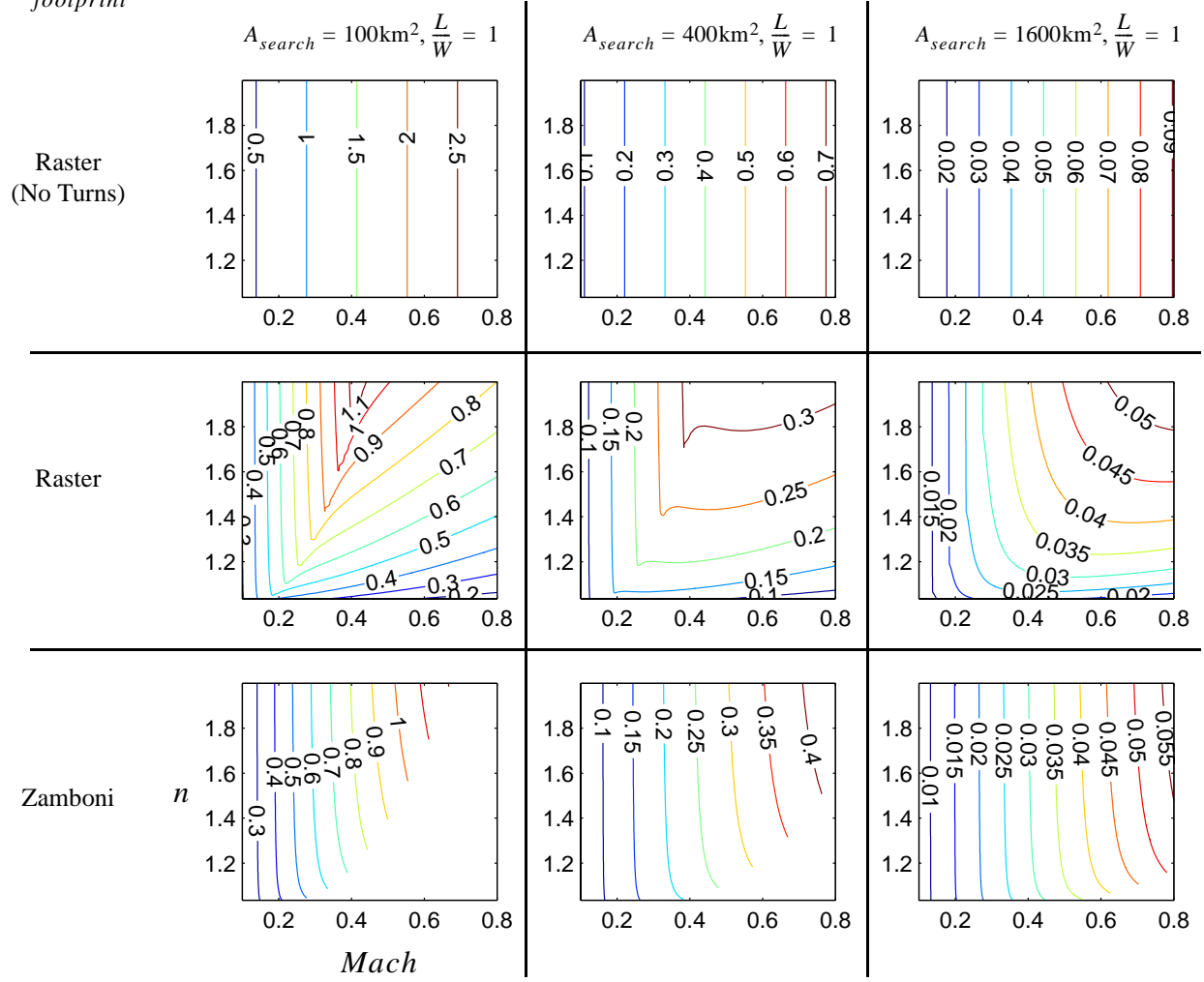
$A_{footprint} = 4\text{km}^2$

Figure 11: Opportunity - Area Search, Search Pattern Comparison, Large Footprint
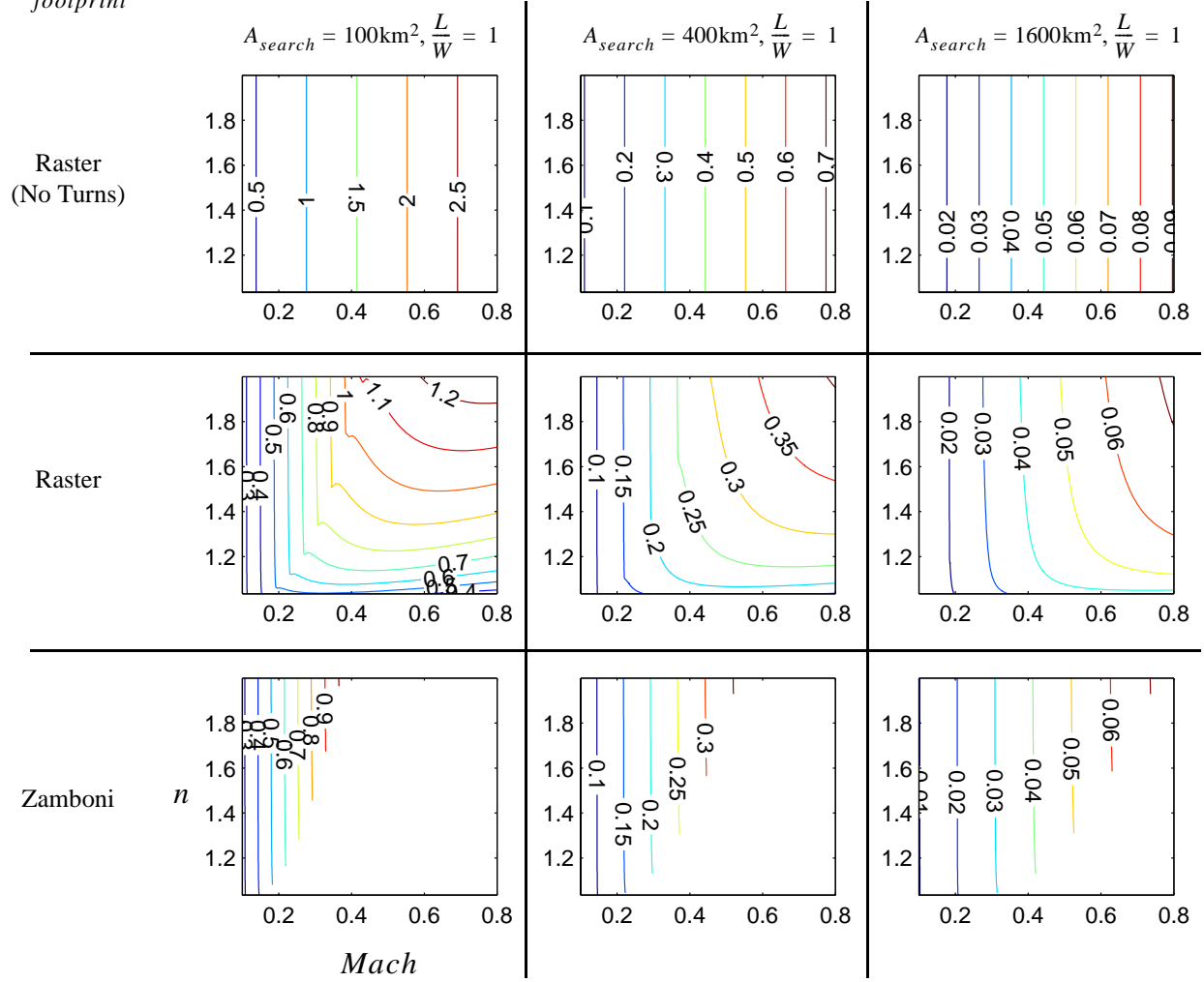
18

$A_{footprint} = 4\text{km}^2$

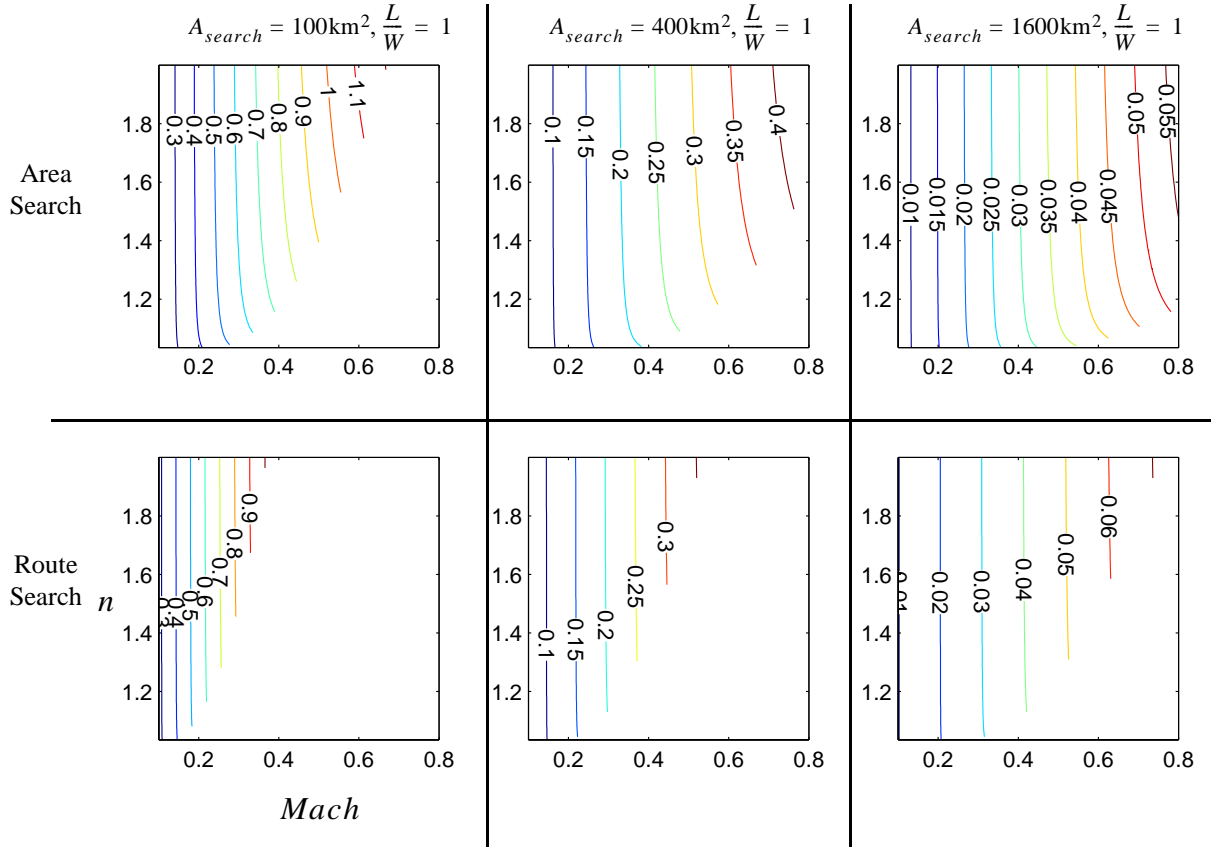Figure 12: Opportunity - Route Search, Search Pattern Comparison, Large Footprint

Figure 13: Opportunity - Raster Search, Area/Route Search Comparison, Large Footprint

Figure 14: Opportunity - Zamboni Search, Area/Route Search Comparison, Large Footprint

5. PROBABILITY-OF-OPPORTUNITY WITH MULTIPLE SENSORS

In the previous section, a single sensor was analyzed for opportunities. For a relatively small area and short Target Availability, a high Probability-of-Opportunity is possible. However, no amount of speed and load factor can make the Probability-of-Opportunity relatively high when the area is large. In this situation, additional sensors are required.

The calculation of the Probability-of-Opportunity depends on how the additional sensors cover the killbox, however. There are several basic possibilities:

1. Multiple sensors on a single vehicle
2. Multiple aircraft flying independent orbits in the same killbox
3. Multiple aircraft each flying one portion of the killbox
4. Multiple aircraft in trail, i.e. flying the same path but separated by a specified period of time
5. Multiple sensors looking at different spots on the ground

Cases 1 and 2 are a simple cumulative probability calculation. For each sensor s with a Probability-of-Opportunity $P_{opp,\,s}$,

$$P_{opp} \;=\; 1 - \prod (1 - P_{opp,\,s}) \qquad\qquad (32)$$

Case 3 is also relatively simple. N aircraft each cover a given portion of the killbox so that there are N smaller killboxes with area A_i. The probability that a target is in a given area is

$$P \;=\; \frac{A_i}{\sum A_i} \qquad\qquad (33)$$

For equal sized areas this reduces to $P = 1/N$. The Probability-of-Opportunity for N aircraft is

$$P_{opp} \;=\; \frac{1}{A} \sum (P_{opp,\,i} A_i) \qquad\qquad (34)$$

For equal size areas with equally performing aircraft,

$$P_{opp} \;=\; P_{opp,\,i} \qquad\qquad (35)$$

In other words, for Case 3 with equal size areas and equally performing aircraft, the total Probability-of-Opportunity is the Probability-of-Opportunity of one aircraft flying in the smaller killbox.

Case 4 requires revisiting the single vehicle and single sensor opportunity derivation. Recall eq. 5.

$$-T_{dwell} < t_{dwell} - t_{target} < T_{target} \tag{36}$$

For an aircraft in trail, the beginning of the dwell time is adjusted by T_{trail}.

$$t_{dwell,2} = t_{dwell,1} + T_{trail} \tag{37}$$

Therefore the modified condition becomes

$$-T_{dwell} - T_{trail} < t_{dwell,2} - t_{target} < T_{target} - T_{trail} \tag{38}$$

and the Probability-of-Opportunity for the aircraft in trail becomes

$$P_{opp,2} = F(T_{target} - T_{trail}) - F(-(T_{dwell} + T_{trail})) \tag{39}$$

The total Probability-of-Opportunity for both aircraft is not the sum of the individual probabilities. Any areas of overlap (intersections) in the probabilities have to be subtracted off. This can best be seen with the PDF in Figure 15 (assuming the dwell times are the same). In more mathematical terms and for two aircraft only (an extension to N aircraft can be made from the two-aircraft case),

$$P_{opp} = P_{opp,1} + P_{opp,2} - (P_{opp,1} \cap P_{opp,2}) \tag{40}$$

An intersection in the probabilities occurs if $T_{trail} < T_{target} + T_{dwell}$.
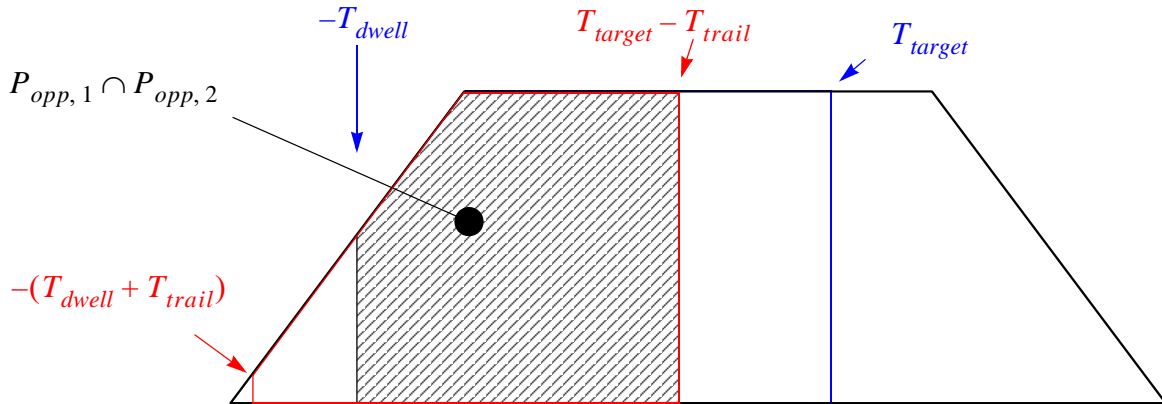
Figure 15: Probability Overlap for Aircraft in Trail

Figure 16 attempts to explain this overlap physically and in terms of time. From the graphic representation, it appears that the interpretation of the mathematical intersection is when both sensors have a chance of looking at the target simultaneously.
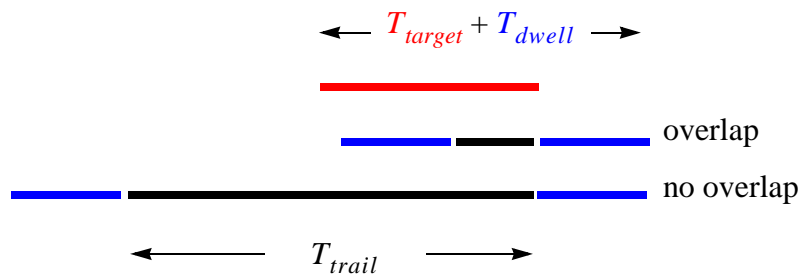

Figure 16: Interpretation of the Intersection of Probability Functions for Aircraft in Trail

So when $T_{trail} > T_{target} + T_{dwell}$, in other words when the trail time is relatively large,

$$P_{opp} = P_{opp,\,1} + P_{opp,\,2} \qquad (41)$$

6. SUMMARY AND CONCLUSIONS

The Probability-of-Opportunity concept developed and explored in this document provides a quick analytic method to explore target and sensor interactions for a generlized search problem. An analytic algorithm was developed, and the relationship was further simplified based on inspection (Figure 6) and a relative magnitude comparison of the remaining terms for realistic search parameters. The final approximate form of the Probaiblity-of-Opportunity is

$$P_{opp} \approx \frac{T_{target}}{T_{revisit}}$$

(42)

which is intuitively correct.

The Opportunity concept requires additional analysis in the form of a detection algorithm to determine the System Probability-of-Find. Because the detection algorithm is specific to a given sensor, this part of the trade space was not explored here. However, the detection algorithms may be sensitive to patrol speed. Therefore, positive trends indicated in this document need to be mitigated until a detection analysis is performed for a particular sensor.

In general, Zamboni searches are better for areas without a long dimension. Rasters are better for narrow search areas such as a route patrol. The Zamboni benefits from more speed and is relatively insensitive to the design sustained load factor. However, the effectiveness of raster searches is highly dependent on speed *and* design load factor.
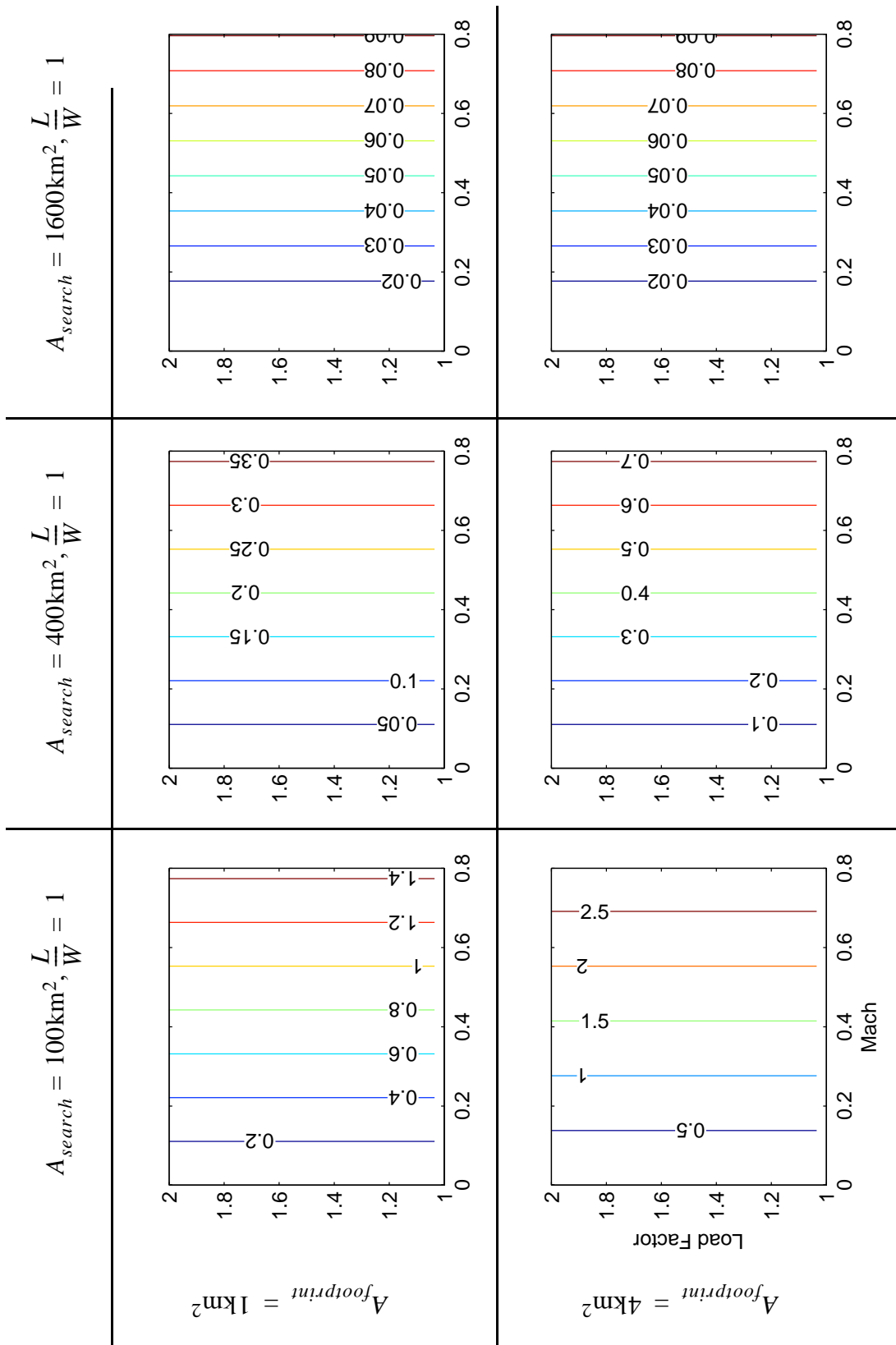
APPENDIX - DETAILED OPPORTUNITY PLOTS

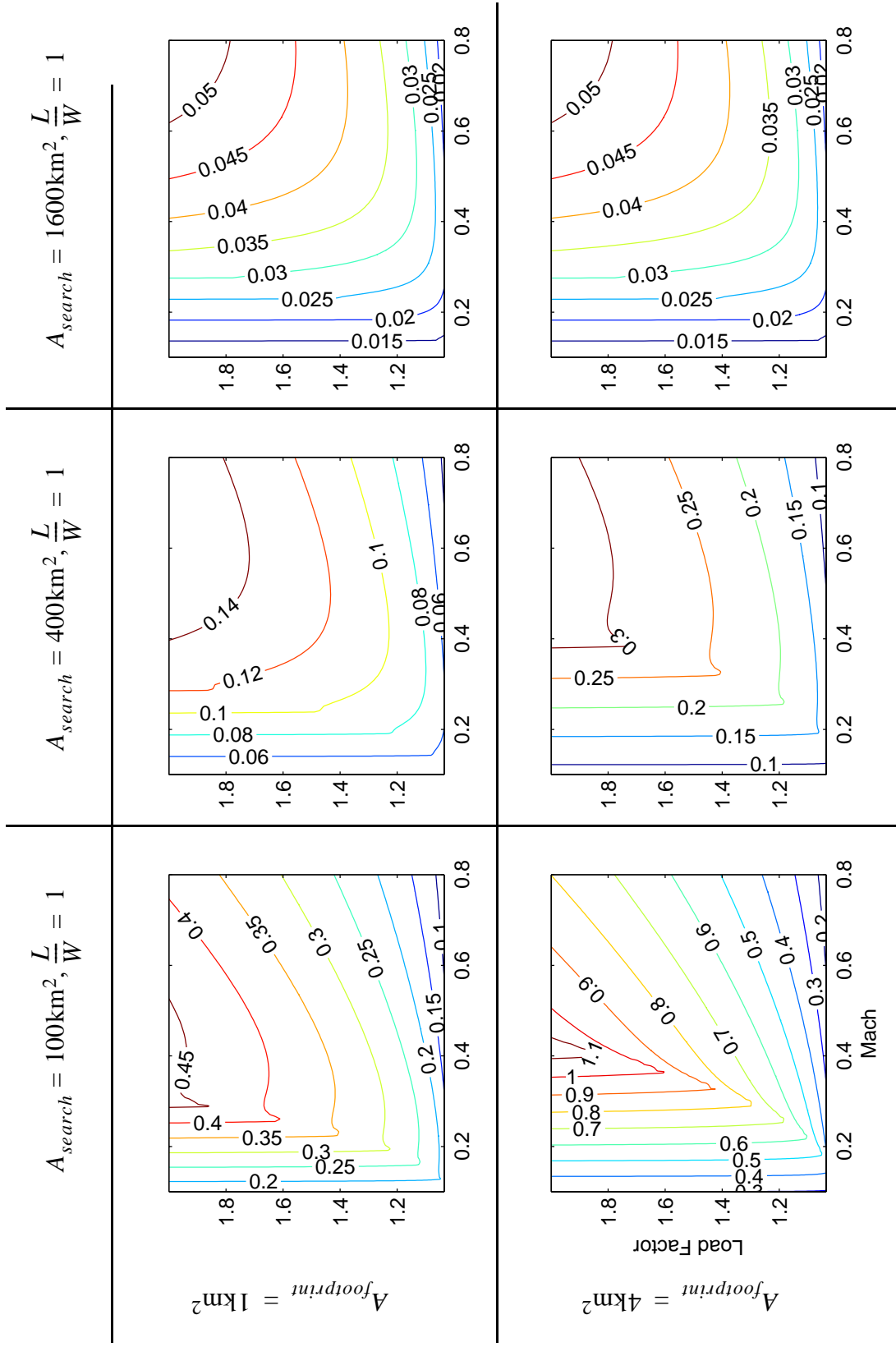**Figure 17: Availability-to-Revisit Time Ratio, Raster Search (No Turns), Square Search**

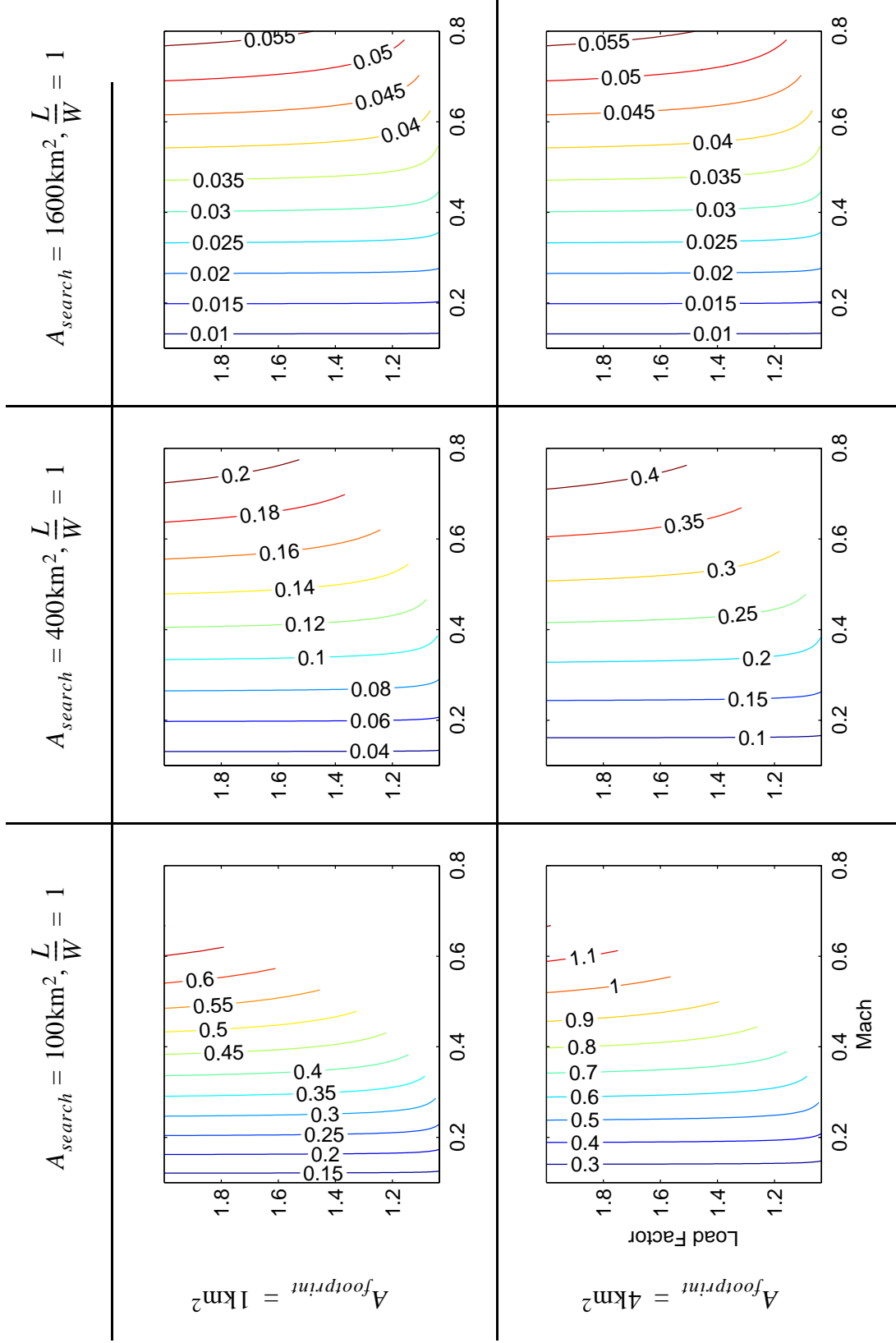**Figure 18: Availability-to-Revisit Time Ratio, Raster Search, Square Search Area**

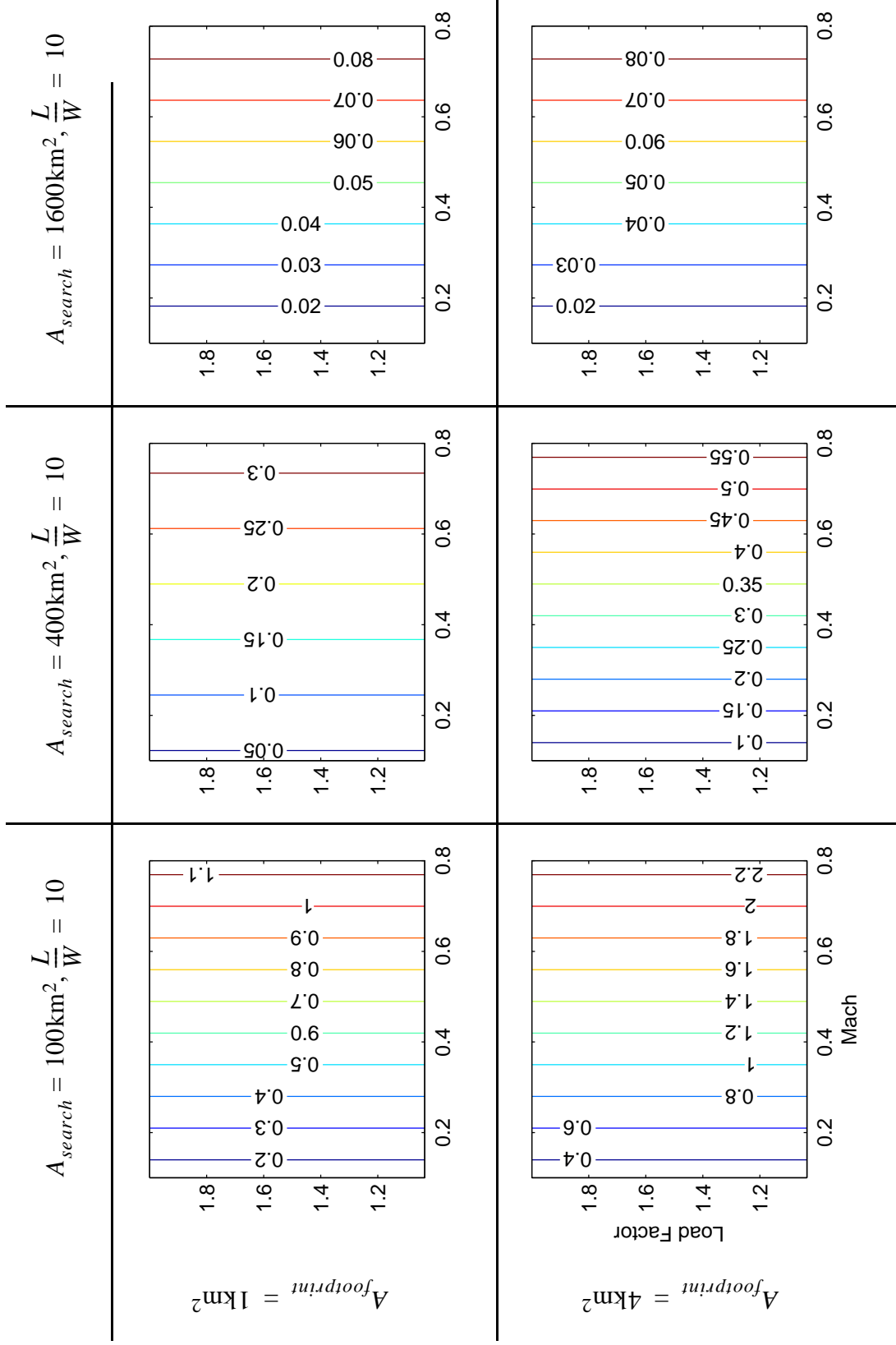**Figure 19: Availability-to-Revisit Time Ratio, Zamboni Search, Square Search Area**

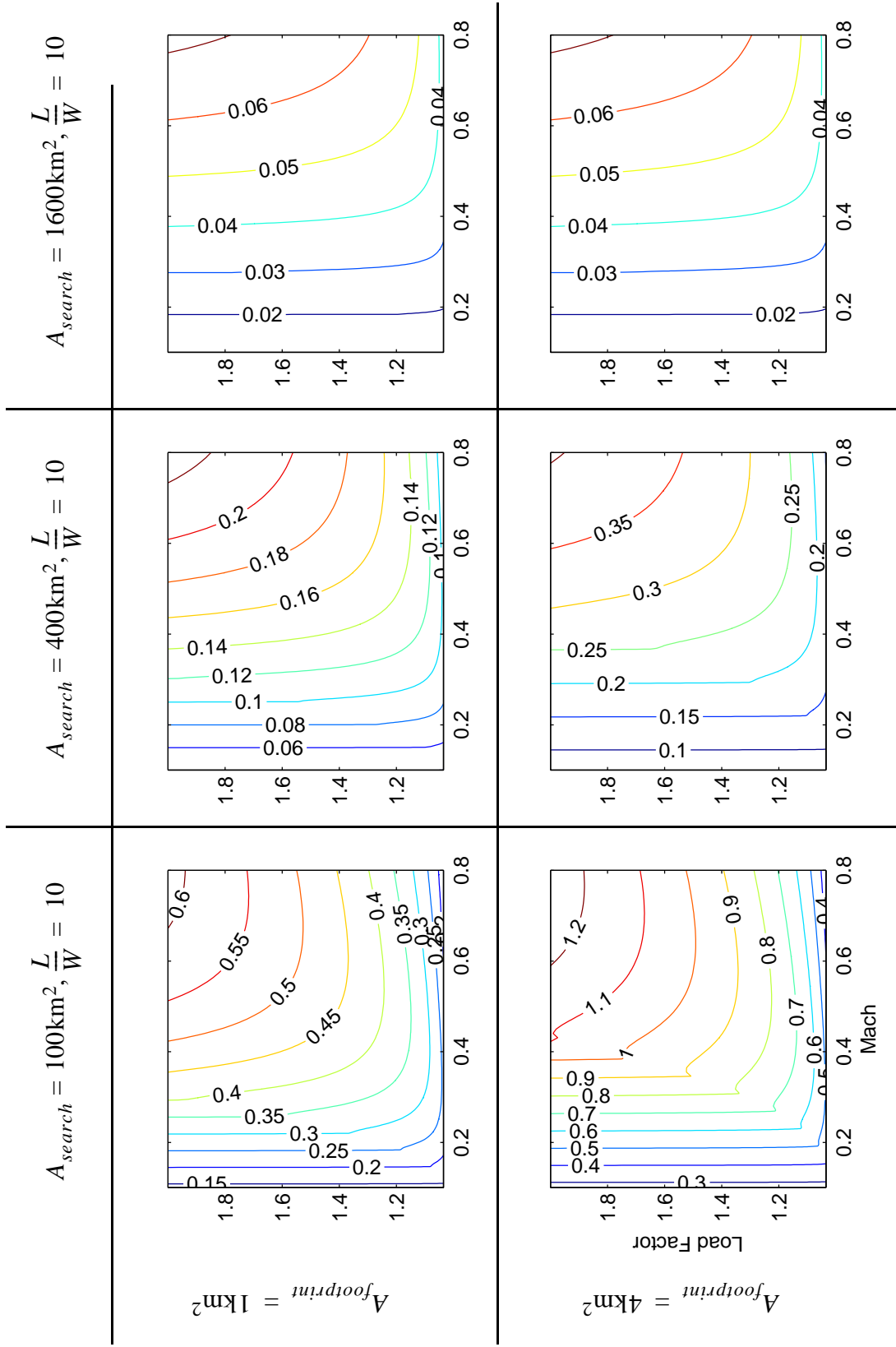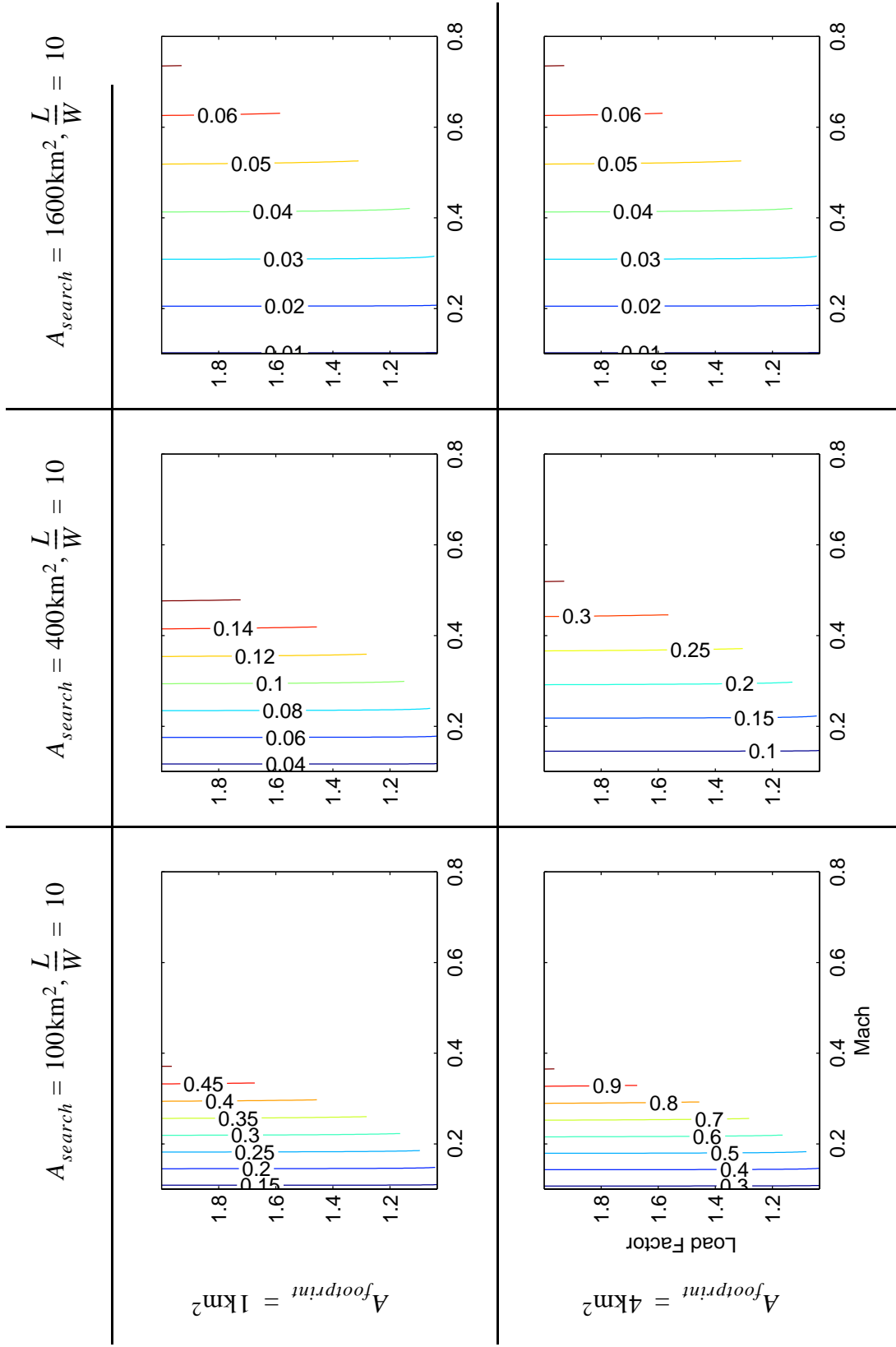**Figure 20: Availability-to-Revisit Time Ratio, Raster Search (No Turns), Route Patrol**

Figure 21: Availability-to-Revisit Time Ratio, Raster Search, Route Patrol

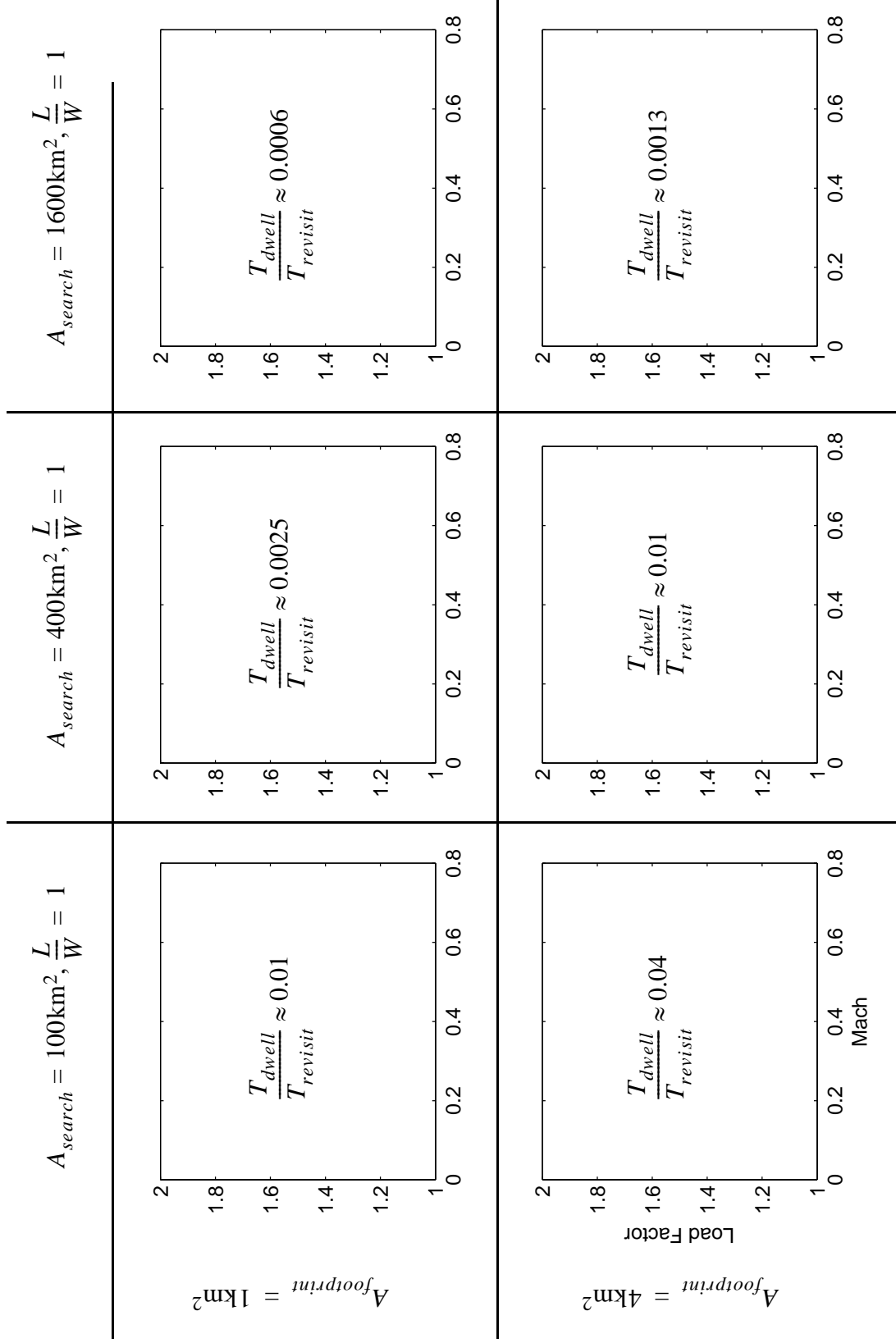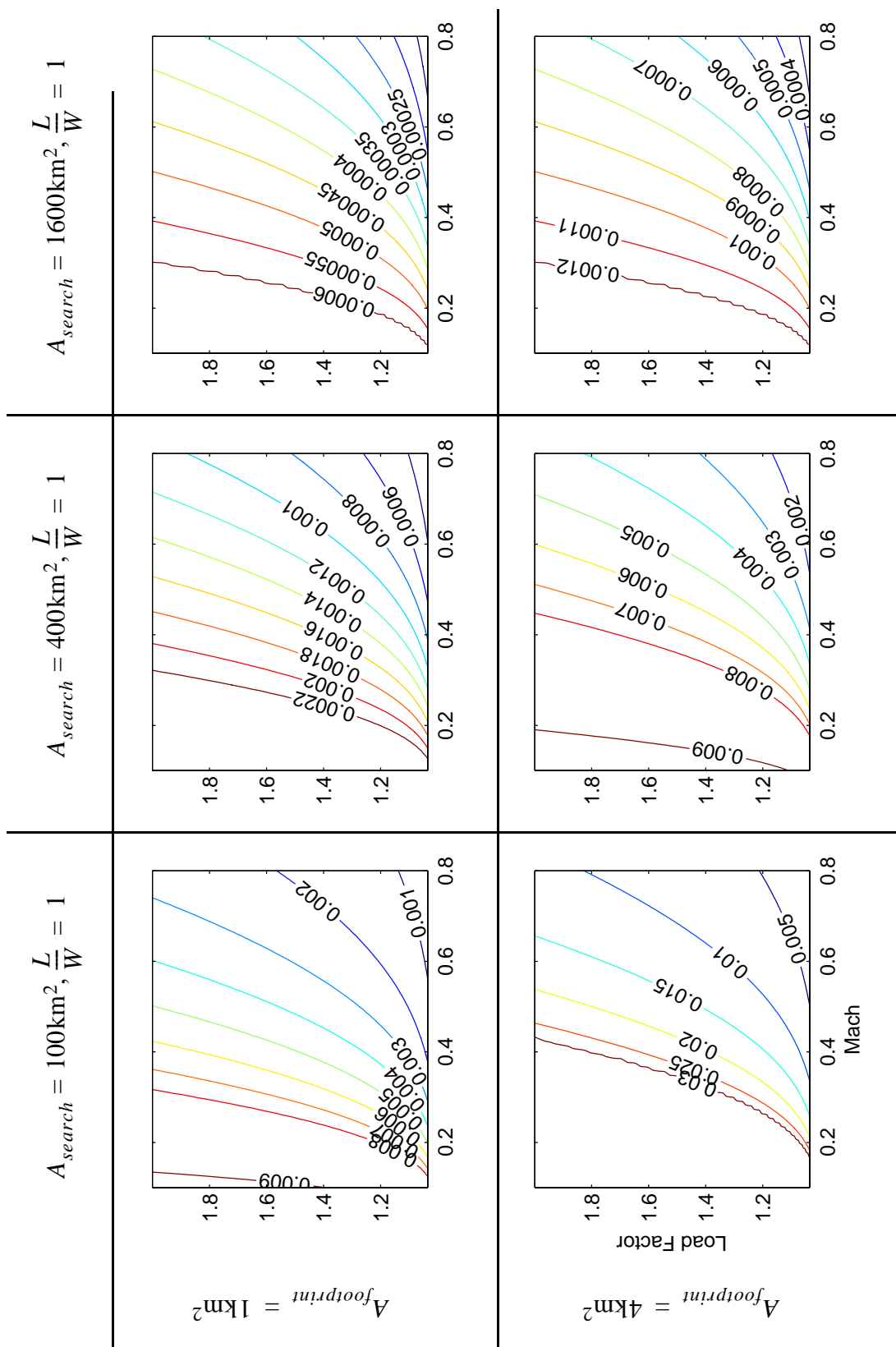**Figure 22: Availability-to-Revisit Time Ratio, Zamboni Search, Route Patrol**

Figure 23: Dwell-to-Revisit Time Ratio, Raster Search (No Turns), Square Search Area

Figure 24: Dwell-to-Revisit Time Ratio, Raster Search, Square Search Area
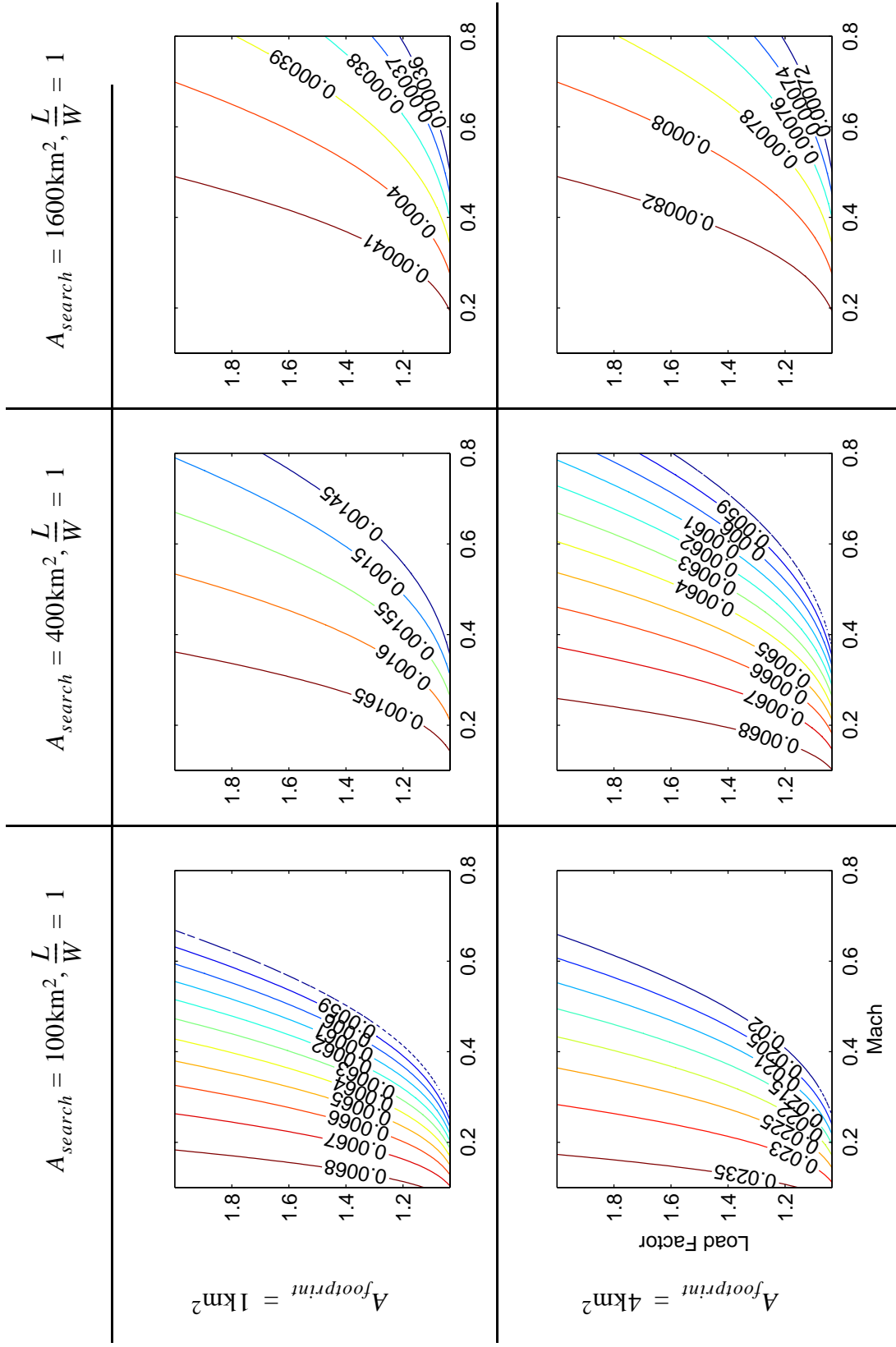
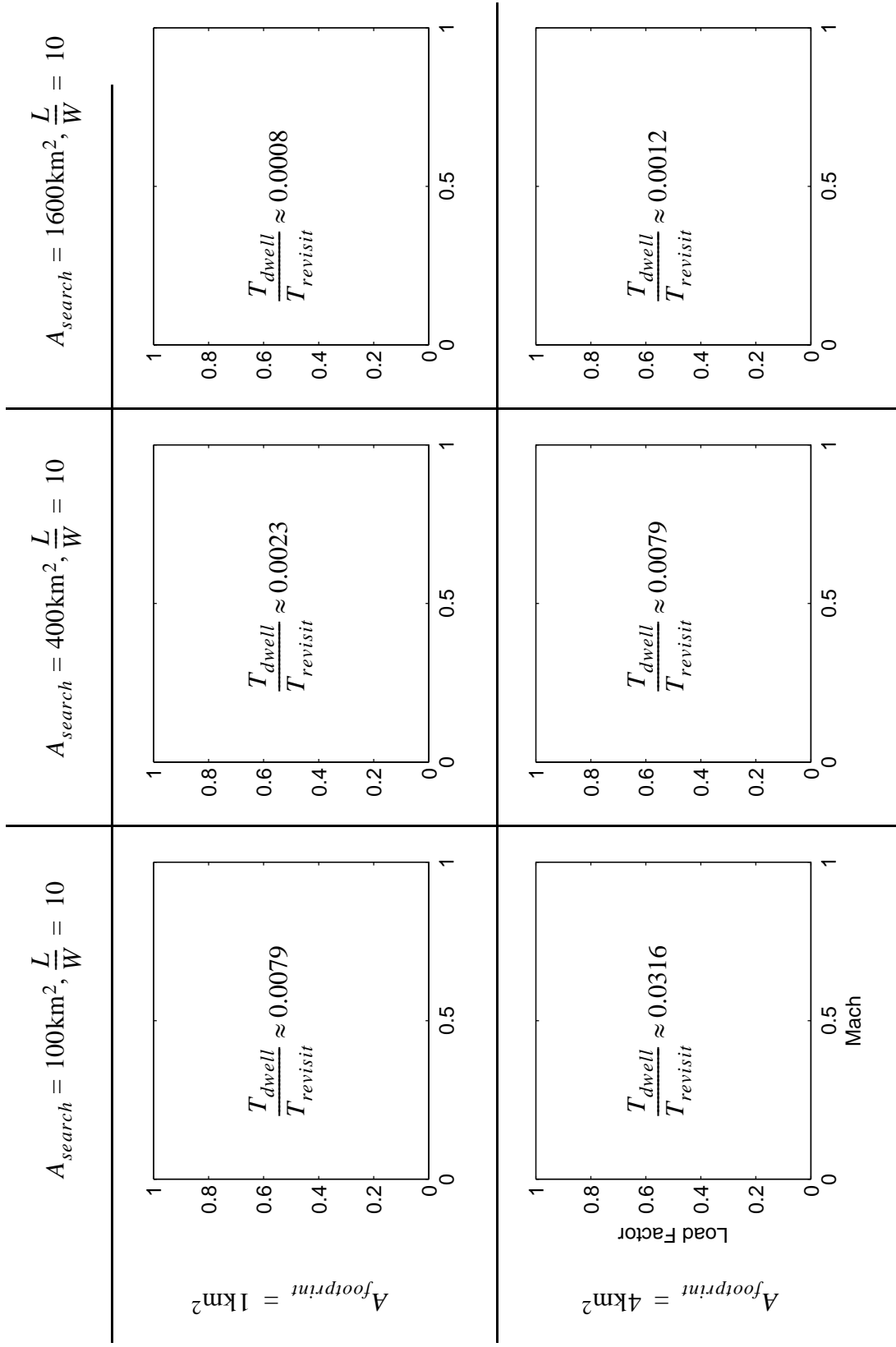**Figure 25: Dwell-to-Revisit Time Ratio, Zamboni Search, Square Search Area**

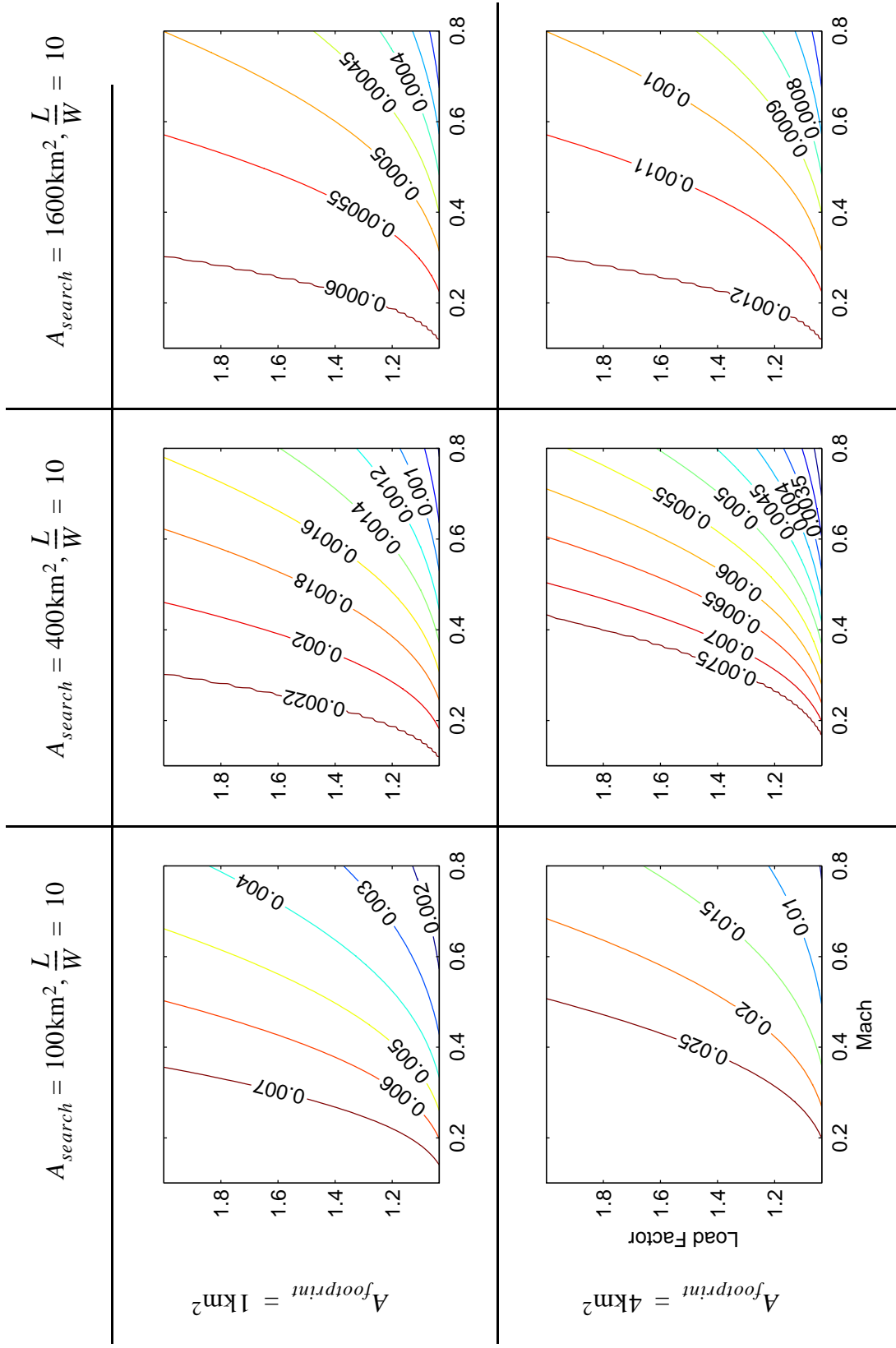**Figure 26: Dwell-to-Revisit Time Ratio, Raster Search (No Turns), Route Patrol**

Figure 27: Dwell-to-Revisit Time Ratio, Raster Search, Route Patrol

Figure 28: Dwell-to-Revisit Time Ratio, Zamboni Search, Route Patrol